HEAT TRANSFER UNDER CONDITIONS OF NUCLEATE BOILING

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New physical concepts are developed in connection with the two fundamental processes resulting in the transfer of heat from a heater to a liquid under conditions of nucleate boiling.

We know from [1, 2] that no satisfactory theory currently exists for heat transfer in nucleate boiling. Moreover, it is presently not clear how the heat is transferred from a hot solid surface to the boiling liquid. In most references devoted to heat transfer in nucleate boiling, the authors deal with their semiempirical formulas by the methods from the theory of similarity [3–11].

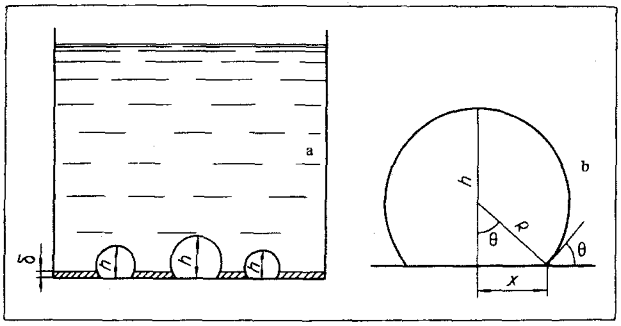

Fig. 1. Vapor bubbles on heater surface.

In this article—partially developing the concepts of [12]—an attempt is undertaken to provide a qualitative picture for the initial stage in nucleate boiling, and also to suggest means of constructing the quantitative physical theory of the heat transfer which takes place in this process.

The heater surface is primarily in contact with the liquid medium in the nucleate boiling regime. Because of the low thermal conductivity of steam, we can neglect the heat transmitted directly to the bubbles, and we can assume that the density of the heat flow in the nucleate boiling regime, just as in the absence of phase conversion, is defined by the conventional formula

$$q = -\lambda \text{ grad } T. \tag{1}$$

The intensity of heat transfer is thus determined by the very temperature gradient which exists within the liquid at the boundary with the heater. In turn, the quantity grad T is a function of the nature of the heat-transport process within the liquid.

There evidently exists a simple relationship between the temperature gradient at the heater surface and the thickness δ of the superheated layer:

$$\text{grad } T = \frac{\Delta T}{\delta}. \tag{2}$$

Using the familiar relationship for the heat-transfer coefficient

$$\alpha = \frac{q}{\Delta T},$$

we obtain the equation

$$\alpha = \frac{\lambda}{\delta}, \tag{3}$$

which establishes that the heat-transfer intensity in nucleate boiling (at a distance from the cris point) is inversely proportional to the thickness of the wall layer for the given liquid. The quantity δ is determined by the rate of heat transport from the superheated layer to the core of the liquid and to the ambient medium. The heat from the wall layer is removed by conduction, natural convection, the evaporation of the superheated liquid in the bubbles situated at the surface, and by forced convection (mixing), this last brought about by the growth, separation, rise, and bursting of the bubbles. As demonstrated by experiment, since the transfer of heat in boiling is considerably greater than the loss of heat in the comparable case in which the liquid fails to boil at identical temperatures, i. e., when the transfer of heat is accomplished exclusively by conduction and natural convection, we are justified in treating the vapor formation and bubbling as fundamental processes of heat transfer in the phase conversion of a liquid into steam. As regards the fraction of heat transferred in each of these processes, this is a function of the boiling conditions: the dimensions, orientation, wettability, and the degree of roughness for the solid heater surface, as well as of the viscosity, the heat of vapor formation, the density, and the surface tension of the liquid. For purposes of a more penetrating analysis into the relation-

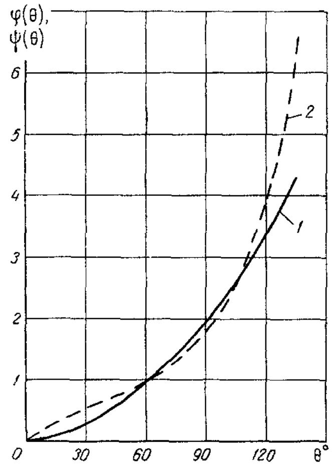

Fig. 2. Plots of φ and ψ as functions of edge angle θ.

ship between the magnitude of heat transfer and the conditions of the process, we will therefore assume the coefficient α to be composed of two terms:

$$\alpha = \alpha_1 + \alpha_2. \tag{4}$$

The quantity α_1 characterizes the intensity of heat transfer due to evaporation; α_2 denotes the heat-transfer intensity brought about by bubbling.

The phenomena responsible for these two heat-transfer processes are different in their physical nature; the relationship between these factors and the conditions of boiling therefore do not coincide. Apparently, the difficulties encountered in deriving an analytical expression for the heat-transfer coefficient as a function of super-heating and of similar parameters arise primarily because this circumstance has been neglected.

The form of the function $\alpha_1(\Delta T)$ can be determined in general form in the following manner. We will assume that the average height $\bar{h}$ of the surface bubbles is substantially greater than the thickness δ of the superheated layer (Fig. 1a). It is natural to assume in this case that the bubbles are in mechanical and thermal equilibrium with the entire thickness of the liquid, i.e., the pressure and temperature within the bubbles are, respectively, denoted by p_0 and T_0. As regards molecular-kinetic equilibrium, this phenomenon is naturally absent with respect to the superheated layer from which the liquid is evaporated into the bubbles. The amount of vapor entering a bubble is proportional to the pressure difference $\Delta p = p_1 - p_0$.

Since the vapor enters a bubble only from the superheated layer, the portion of the heat flow q_1 which is extended on evaporation must be proportional to the specific heat of vapor formation r, the density of the active centers n, the pressure difference Δp, and the mean length $2\pi\bar{x}$ of the perimeter of the surface-bubble base:

$$q_1 = \text{const}\, rn\bar{x}\Delta p. \tag{5}$$

For superheating that is not too extensive, we are correct in assuming that

$$\Delta p = \frac{dp}{dT}\Delta T.$$

Consequently, for the coefficient α_1 we have the relationship

$$\alpha_1 = \text{const}\, rn\bar{x}\frac{dp}{dT}. \tag{6}$$

The bubble-base radius x is easily calculated if we neglect the deviation of bubble shape from the spherical. As is clear from Fig. 1b, here

$$x = R\cos\theta.$$

The spherical radius R_{max} of the maximum bubble can be found by comparing the volume of the segment

$$V_{max} = \frac{1}{3}\pi R_{max}^3 (2 + 3\cos\theta - \cos^3\theta)$$

with the separation volume of the bubble

$$V_0 = \frac{4}{3}\pi R_0^3,$$

where, according to the Fritz formula,

$$R_0 = \text{const}\ \theta\left(\frac{\sigma}{\rho g}\right)^{1/2}. \tag{6'}$$

Denoting $f(\theta) = (2 + 3\cos\theta - \cos^3\theta)/4$ and introducing the concept of the mean radius $\bar{R}$, we obtain

$$\bar{R} = \frac{1}{2}R_{max} = \frac{R_0}{2\sqrt[3]{f(\theta)}}.$$

This gives us the following:

$$\bar{x} = \frac{R_0\cos\theta}{2\sqrt[3]{f(\theta)}}. \tag{6''}$$

Finally, having substituted the value of R_0 from (6') into this last expression, we bring it to the form

$$\bar{x} = \text{const}\,\varphi(\theta)\left(\frac{\sigma}{\rho g}\right)^{1/2}, \tag{7}$$

where $\varphi(\theta) = \theta\cos\theta\,[f(\theta)]^{-1/3}$ is a function of θ, whose curve is shown in Fig. 2.

Now, to determine the nature of the relationship between α_1 and the superheating, let us examine the manner in which ΔT influences each of the factors in the right-hand member of (6). In the case of low superheating, the average bubble dimension, the specific heat of vapor formation, and the derivative dp/dT can be assumed constant in approximate terms, and conversely, the number of active centers is an extremely strong function of the superheating, and namely [12],

$$n = \text{const}\,(\Delta T)^2. \tag{7'}$$

Having substituted (7) and (7') into (6), we obtain an equation valid for the first stage of nucleate boiling:

$$\alpha_1 = \text{const}\, r\frac{dp}{dT}(\Delta T)^2\varphi(\theta)\left(\frac{\sigma}{\rho g}\right)^{1/2}. \tag{8}$$

In addition, if we use the Clapeyron-Clausius formula

$$\frac{dp}{dT} = \frac{r\rho}{T_s},$$

the expression for the first coefficient of heat transfer assumes its final form

$$\alpha_1 = ar^2\varphi(\theta)\frac{(\rho\sigma)^{1/2}}{T_s g^{1/2}}(\Delta T)^2. \tag{9}$$

Here a is a constant characterizing the state (roughness) of the horizontal heating surface.

To determine the form of the function $a_2(\Delta T)$, we should bear in mind that it is only the motion of the surface bubbles that is significant in the removal of heat from the thin wall layer, and this is accomplished by mixing. Indeed, as the "three-dimensional" bubble separates from the heater surface and enters the main part of the liquid, floating upward, it brings about the mixing of the liquid, primarily in the vicinity of the adjacent layer. Since the surface bubbles are surrounded at the base by the liquid superheated at the wall, during their growth, as well as at the instant of separation, the bubbles can cause the displacement and the mixing of precisely this superheated layer.

It is difficult for the surface bubble to move as an entire whole, since it is attached to the surface of a solid along which it is impossible for the liquid to slip. The base of a growing bubble therefore does not change smoothly, but rather in jumpwise fashion; the shape of the bubble deviates from the equilibrium shape in this event. This leads to fluctuations in the shapes of the surface bubbles. The faster their growth, the more intensive their fluctuation. The existence of such fluctuations has been confirmed in our laboratory by T. S. Chigareva.

It is our contention that it is these pulsating surface bubbles that are primarily responsible for the mixing of the liquid in the wall layer and lead to the intensification of convective heat transfer. A definite confirmation of this assumption is the familiar fact of improvement in heat transfer when the wall layer of a boiling liquid is subjected to ultrasonic treatment [13]. The surface bubbles begin to pulsate intensively under the action of the ultrasonic field, thus setting up vortex motion and the mixing of the liquid in the superheated layer, as a result of which the transfer of heat from the heater is intensified.

Proceeding from general physical considerations, we can qualitatively evaluate α_2 as a function of the superheating and of other process parameters. We know that the oscillations of a body emersed into an actual liquid bring about the vortex motion of the adjacent layers, with the depth of penetration for this motion being of the order of magnitude [14]

$$l \sim \sqrt{\frac{\eta}{\rho\omega}}, \tag{10}$$

where ω is the cyclical oscillation frequency. It is natural to assume that the quantity of heat transported by mixing per unit time per unit area of heater surface is proportional to the number n of bubbles, the average amplitude A of their pulsations, the depth of penetration l, the specific heat capacity c of the liquid, and the temperature difference $\Delta T = T_1 - T_0$:

$$q_2 = \text{const } ncAl\,\Delta T. \tag{11}$$

The natural frequency of the pulsations executed by the three-dimensional bubbles in the liquid is determined by the relationship [14]

$$\omega = \sqrt{\frac{8\sigma}{\rho R^3}}. \tag{12}$$

As demonstrated by T. S. Chigareva, in the case of surface bubbles, instead of (12) we obtain a slightly altered formula:

$$\omega = \sqrt{\frac{6\sigma(1+\cos\theta)}{\rho R^3}}. \tag{12'}$$

Substituting (12') into (10), we find that l is directly proportional to the mean linear dimension of the bubble $R = R_{max}/2$ in a ratio of 3 : 4.

For the second coefficient of heat transfer we thus obtain

$$\alpha_2 = \text{const } ncA\,\eta^{1/2}\,R_0^{3/4} \times$$
$$\times [\rho\sigma(1+\cos\theta)]^{-1/4}[f(\theta)]^{-1/3}. \tag{13}$$

The constants c, ρ, η, and σ for the liquid can, in first approximation, be regarded as independent of the temperature difference ΔT. The relationship between n and ΔT is obtained from Eq. (7'). As regards the amplitude A, it is the more substantial, the greater the difference Δp between the internal and external bubble pressures. We are therefore correct in writing

$$A = \text{const}\,\Delta p = \text{const}\frac{r\rho}{T_s}\Delta T. \tag{14}$$

Having substituted into (13) the values of n and A from (7') and (14), as well as the separation radius R_0 from the Fritz formula (6'), from the second heat-transfer coefficient we obtain

$$\alpha_2 = b\,\frac{rc}{T_s}\,\psi(\theta)\left(\frac{\eta^2\sigma^{1/2}\rho^{11/2}}{g^{3/2}}\right)^{1/4}(\Delta T)^3,$$
$$\psi(\theta) = \theta^{3/4}(1+\cos\theta)^{-1/4}[f(\theta)]^{-1/3}, \tag{15}$$

where the factor b in this stage can be determined only by experimentation. The consolidation of (9) and (15) gives us a relationship for the total heat-transfer coefficient in the first stage of nucleate boiling (the solitary-bubble regime)

$$\alpha = \frac{r\rho^2}{T_s g^{1/2}} \times$$
$$\times \{ar\varphi(\theta)\sigma^{1/2}\rho^{-3/2} + bc\psi(\theta)\nu^{1/2}\sigma^{1/8}\Delta T\}(\Delta T)^2. \tag{16}$$

Despite the cumbersome nature of this formula—which, moreover, includes indeterminate parameters *a* and b—it makes it possible for us to derive significant information regarding the effect of the heat of vapor formation, of the edge angle, the density, viscosity, acceleration of the force of gravity, the heat capacity, and similar characteristics of the liquid on the intensity of heat transfer in the case of nucleate boiling.

NOTATION

λ is the thermal conductivity of the liquid; c is the specific heat capacity of the liquid; ρ is the liquid density; r is the specific heat of vapor generation; n is the density of the active centers; T_1 and T_s are the temperatures of the heater and the liquid (saturation) core; ΔT is the temperature head; σ is the surface tension factor; θ is the edge angle; η is the liquid viscosity; ν is the kinematic viscosity; g is the gravitational acceleration; p is the external pressure; p_1 is the elasticity of the saturated vapor at T_1; ω and A are the cyclic frequency and amplitude of bubble pulsation; δ is the thickness of the superheated bed; l is the penetration thickness of eddy motion due to bubble pulsation in the liquid; α is the heat transfer coefficient; x is the base radius of the surface bubble; $\varphi(\theta)$ and $\psi(\theta)$ are the functions of the edge angle θ; R_0 is the separation radius of the bubble; h is the height of the bubble surface.

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